

Answers to Problem Set 7

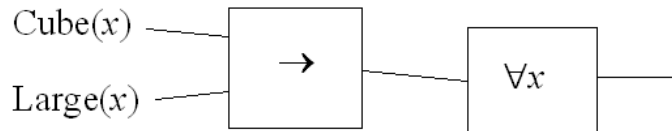
Total: 99 marks (Yeah, no one's getting 100!)

1. [2 marks each]

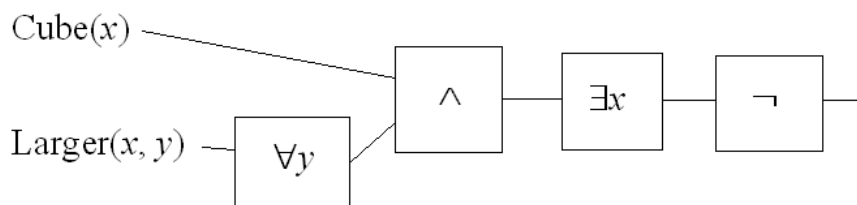
(i)	$\forall a(\text{Large}(a) \rightarrow \text{Cube}(a))$	Not a wff.
(ii)	$\text{Larger}(\exists, b) \rightarrow \text{Tet}(b)$	Not a wff.
(iii)	$\exists z(\text{Cube}(z) \rightarrow \text{Tet}(z))$	Wff + Sentence.
(iv)	$\text{Cube}(a) \rightarrow \text{Tet}(x) \rightarrow \text{Tet}(y)$	Not a wff.
(v)	$\exists x(\text{Dodec}(x) \wedge \text{Large}(\mathfrak{y}))$	Wff but not a sentence
(vi)	$\forall z(\exists w \text{ Large}(w) \rightarrow \text{Cube}(z))$	Wff + Sentence
(vii)	$\exists t \text{ Cube}(t) \rightarrow \forall t \text{ Cube}(t)$	Wff + Sentence
(viii)	$\forall v \text{ Cube}(v) \rightarrow \text{Large}(\mathfrak{y})$	Wff but not a sentence
(ix)	$\forall x(\text{Dentist}(x) \rightarrow \exists y(\text{Larger}(y, x) \wedge \forall z(\text{Between}(z, \text{mother}(y), x) \rightarrow \text{Tet}(z))))$	Wff + Sentence
(x)	$\forall x(\text{Large}(\text{Cube}(x)) \rightarrow \text{Small}(y))$	Not a wff.

1. [2 marks for each sentence/diagram, 1 for main operator, 15 total]

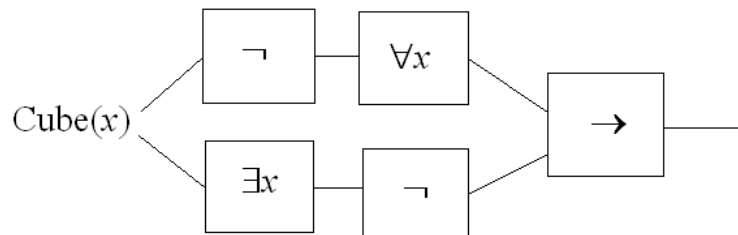
(i) *Universal Sentence*. [Question was: $\forall x(\text{Cube}(x) \rightarrow \text{Large}(x))$]



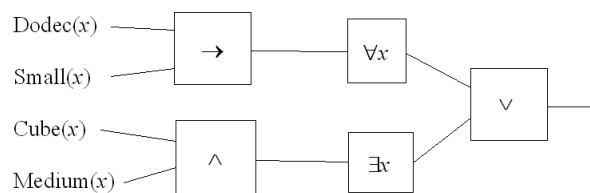
(ii) *Negation Sentence* [Question was: $\neg \exists x(\text{Cube}(x) \wedge \forall y \text{ Larger}(x, y))$]



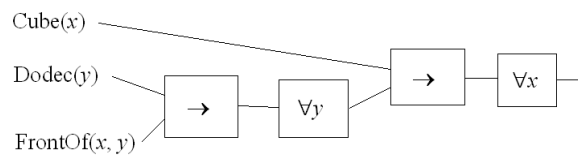
(iii) *Conditional Sentence* [Question was: $\forall x \neg \text{Cube}(x) \rightarrow \neg \exists x \text{ Cube}(x)$]



(iv) *Disjunction Sentence* $\forall x(\text{Dodec}(x) \rightarrow \text{Small}(x)) \vee \exists x(\text{Cube}(x) \wedge \text{Medium}(x))$



(v) *Universal Sentence* $\forall x(\text{Cube}(x) \rightarrow \forall y(\text{Dodec}(y) \rightarrow \text{FrontOf}(x, y)))$



3. [1 mark each]

F 1. $\forall x (x = a \vee x = b \vee x = c \vee x = d)$

T 2. $\exists x (x \neq a \wedge x \neq b \wedge x \neq c \wedge x \neq d \wedge x \neq e)$

T 3. $\forall x (x = a \rightarrow x = d)$

F 4. $\exists x (\text{Between}(x, c, a) \wedge x \neq b)$

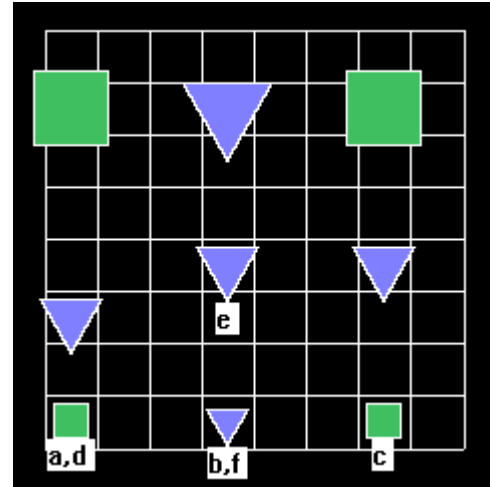
T 5. $\forall x (\text{Between}(x, c, a) \rightarrow x = b)$

F 6. $\forall x ((\text{Tet}(x) \wedge \text{Medium}(x)) \rightarrow x = e)$

T 7. $\forall x (x = e \rightarrow (\text{Tet}(x) \wedge \text{Medium}(x)))$

T 8. $\forall x ((\text{Tet}(x) \wedge \text{Small}(x)) \leftrightarrow x = b)$

T 9. $\exists y (y \neq e \wedge \text{SameRow}(y, e))$



9.9 [5 marks for world]

T 1. $\exists x (\text{Tet}(x) \wedge \text{Large}(x))$

T 2. $\exists x (\text{Tet}(x) \wedge \text{Medium}(x))$

T 3. $\exists x (\text{Cube}(x) \wedge \neg \text{Small}(x))$

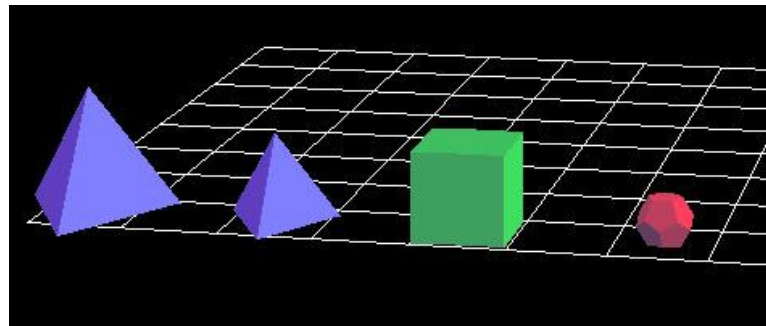
T 4. $\exists y (\text{Dodec}(y) \wedge \neg \text{Large}(y))$

T 5. $\forall x (\text{Cube}(x) \rightarrow \text{Medium}(x))$

T 6. $\forall x (\text{Dodec}(x) \rightarrow \text{Small}(x))$

T 7. $\forall x (\text{Tet}(x) \rightarrow \neg \text{Small}(x))$

T 8. $\forall y (\text{Cube}(y) \rightarrow \neg \text{Tet}(y))$



9.12 [1 mark each]

T 1. $\forall x (\text{Tet}(x) \rightarrow \text{Small}(x))$
T 2. $\forall x (\text{Cube}(x) \rightarrow \text{Small}(x))$
T 3. $\forall y (\text{Dodec}(y) \rightarrow (\text{Small}(y) \vee \text{Medium}(y) \vee \text{Large}(y)))$
T 4. $\exists x (\text{Dodec}(x) \wedge \text{Large}(x))$
T 5. $\exists x (\text{Dodec}(x) \wedge \neg \text{Large}(x))$
T 6. $\exists x (\text{Dodec}(x) \wedge \text{Small}(x))$
T 7. $\exists x (\text{Dodec}(x) \wedge \neg \text{Small}(x))$
T 8. $\exists x (\text{Dodec}(x) \wedge \neg (\text{Large}(x) \vee \text{Small}(x)))$
T 9. $\forall x (\text{Tet}(x) \rightarrow \neg \text{Large}(x))$
T 10. $\neg \exists z (\text{Cube}(z) \wedge \text{Large}(z))$

9.18 [2 marks each]

T 1. $\neg \exists x (\text{Cube}(x) \wedge \text{Medium}(x))$
T 2. $\neg \exists x \text{FrontOf}(x, b)$
T 3. $\forall x (\text{Cube}(x) \rightarrow (\text{FrontOf}(x, e) \vee \text{BackOf}(x, e)))$
T 4. $\neg \exists x (\text{Cube}(x) \wedge \text{Between}(x, a, c))$
T 5. $\forall x (\text{SameCol}(x, a) \vee \text{SameCol}(x, b) \vee \text{SameCol}(x, c))$

9.20 [2 marks each]

1. Claire never gave Folly to Max.
2. Every pet is hungry at 2pm.
3. No person owned Pris at 2pm.
4. No angry student fed Carl at 2pm.
5. At 2pm Max gave all the pets he owned to Claire.

9.22 [1 mark each] – total 20

(N.B. The values for Malcev's World and Bolzano's World are included just for interest.)

	Malcev's	Bolzano's	Boole's	Wittgenstein's
1.	F	F	T	FALSE
2.	T	F	F	F
3.	T	F	FALSE	F
4.	T	F	F	F
5.	TRUE	F	T	F
6.	F	F	F	F
7.	F	F	T	F
8.	F	TRUE	T	F
9.	T	T	T	T
10.	T	T	T	T